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A NOTE ON THE USE OF A BOUNDARY-LAYER MODEL FOR CORRELATING FILM-COOLING DATA

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Abstract—The boundary-layer model for correlating experimental data on film-cooled adiabatic walls is re-examined. Existing analyses are reviewed. A much simpler approach is given which can also cover the case of foreign gas injection. The utility and limitations of the boundary-layer model are emphasized by comparison with experimental data.

NOMENCLATURE

A, B, C, constants, see text;
C_f, local skin friction coefficient;
C_p, specific heat at constant pressure;
H, heat-transfer coefficient defined in text;
K, constant, see text;
L, unheated starting length, see equation (10);
M, Mach number;
Pr, Prandtl number;
Re, Reynolds number;
T, temperature;
T^*, reference temperature, see equation (10);
f, function of;
h, enthalpy;
k, coefficient of thermal conductivity;
k_e, eddy heat conductivity;
$\dot{m}$, mass flow rate;
m, $\rho_c u_c/\rho_m u_m$;
n, power law index;
p, pressure;
q, heat-transfer rate;
s, slot height;
t, time;
u, x component of velocity;
v, y component of velocity;
x_0, fictitious starting point of coolant boundary layer, see Fig. 1;
x, y, cartesian co-ordinates;
$\bar{x}$, $(x/ms).\{Re_c.(\mu_c/\mu_m)\}^{-1/4}$;
a, thermal diffusivity, $k/\rho C_p$;
β, eddy diffusivity, $k_e/\rho u C_p$;
α, β, constants, see text;
Γ, gamma function;
δ, boundary-layer thickness;
δ_1, boundary-layer displacement thickness;
δ_2, boundary-layer momentum thickness;
δ_T, boundary-layer thermal thickness;
η, $(T_{aw} - T_m)/(T_c - T_m)$;
η', $(h_{aw} - h_{om})/(h_{oc} - h_{om})$;
η'', $(T_{aw} - T_{om})/(T_{oc} - T_{om})$;
θ, $(T - T_w)/(T_m - T_w)$;
μ, coefficient of viscosity;
ξ, see equation (16);
ρ, density;
σ, $(T - T_m)$;
τ, skin friction.

Suffices

c, coolant;
m, mainstream;
o, total;
s, slot;
w, wall;
aw, adiabatic wall.

INTRODUCTION

WHEN A WALL is film cooled, by injecting a stream of gas between the surface and the hot external flow, three separate regions can be recognized, as shown in Fig. 1. A "potential core", wherein the wall temperature remains close to the coolant gas temperature, is followed by a zone where the velocity profile is similar to that of a wall jet. Farther downstream, however, the flow must become similar to that in a fully developed turbulent boundary layer. For coolant and mainstream gases of similar density the relative length of the three regions is governed primarily by the velocity ratio u_c/u_m. When $u_c \gg u_m$ a simple jet model, as suggested by Spalding [1] for the second zone, may be valuable.

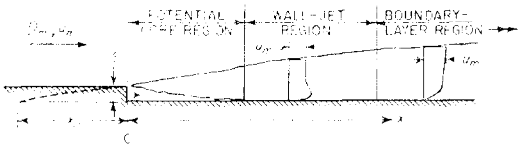

FIG. 1. The three possible regions for the flow past a film cooled surface.

For $u_c < u_m$ the second zone is non-existant and the boundary-layer model for correlating film cooling data should be useful. The re-derivation, critical examination and extension (to cover foreign gas injection and large density ratios, ρ_c/ρ_m) of the boundary-layer model is the subject of this note.

SURVEY OF PREVIOUS THEORETICAL WORK

Wieghardt [2] made one of the first investigations of slot injection in connexion with the problem of de-icing. He achieved an asymptotic solution of the turbulent boundary-layer equations of continuity and energy by assuming similarity for both the velocity and temperature profiles. His solution gave $T(y)$ and $T_{aw}(\delta)$ but not $T_{aw}(x)$. Wieghardt noted that the assumption $(\delta/x) \sim Re_x^{-1/5}$ would indicate $T_{aw} \sim x^{-4/5}$, a result in excellent agreement with his experiments, but he was unwilling to take this step. So although he was almost certainly aware of the simple expression for $T_{aw}(x)$ given in this note, he did not write it down, preferring instead to find $T_{aw}(x)$ experimentally.

Since his work provided a foundation for much that was to follow, it is worth considering in more detail. The basic assumptions in his analysis are:

(i) for $x/s \gg 1$ the flow near the wall resembles the flow in a boundary layer
(ii) low Mach number flow, the dynamic temperature rise is neglected
(iii) C_p is constant throughout the flow
(vi) the molecular heat conductivity is neglected in comparison with the eddy heat conductivity
(v) for large x/s the temperature profiles are similar, hence

$$\frac{\sigma}{\sigma_w(x)} = f\left[\frac{y}{\delta_T(x)}\right]$$

where $\sigma = T - T_m$

and $\delta_T(x) = \int_0^\infty (\sigma/\sigma_w)\, dy$

(vi) the mass-velocity profiles are also similar and may be written in power law form

$$\frac{\rho u}{\rho_m u_m} = \left(\frac{y}{\delta}\right)^n$$

(vii) k is a function of x only.

In the light of these assumptions the relevant equations may be written as

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \text{ continuity} \quad (1)$$

$$\rho u \cdot \frac{\partial \sigma}{\partial x} + \rho v \cdot \frac{\partial \sigma}{\partial y} = k\frac{\partial^2 \sigma}{\partial y^2} \qquad \text{energy} \quad (2)$$

$$\int_0^\infty \rho u \sigma\, dy = \text{const.} = \rho_c u_c \sigma_c s \quad \text{conservation}$$
of energy for the case of an adiabatic wall (3)

By introducing assumptions (v) and (vi) into these equations and applying the boundary conditions

$$\frac{\partial T}{\partial y} = 0, \quad T = T_{aw} \quad \text{at} \quad y = 0$$

$$T = T_m \quad \text{at} \quad y = \infty$$

Wieghardt solves for $\sigma(y)$ obtaining

$$\frac{\sigma}{\sigma_{aw}} = \exp\left[-C_1\left(\frac{y}{\delta_T}\right)^{n+2}\right] \quad (4)$$

where

$$C_1 = \left\{\Gamma\left(\frac{n+3}{n+2}\right)\right\}^{n+2}.$$

Substituting this result in the energy balance equation (3), and defining δ by $y = \delta$ when $\sigma/\sigma_{aw} = 0{\cdot}1$, he finds that for $n = 1/7$

$$\frac{\sigma_{aw}}{\sigma_c} = \frac{T_{aw} - T_m}{T_c - T_m} = \eta = 2{\cdot}01 \frac{ms}{\delta}. \quad (5)$$

Wieghardt points out that further progress is impossible since the variation of δ with x is unknown. However, if the assumption is made that $\delta = 0{\cdot}37\, x\, Re_x^{-1/5}$ then one obtains the result

$$\eta = 5{\cdot}44 \left(\frac{x}{ms}\right)^{-0\cdot8} \left(Re_c \,.\, \frac{\mu_c}{\mu_m}\right)^{0\cdot2}. \quad (6)$$

Hartnett *et al.* [3] adopted Wieghardt's analysis but simplified the mathematics by writing equation (4) as

$$\sigma/\sigma_{aw} = \exp\left[-C(y/\delta)^2\right] \quad \text{since} \quad n \ll 2 \quad (7)$$

Expanding the right-hand side of (7), retaining only the first three terms and substituting in (3) gives

$$\int_0^1 \left(\frac{y}{\delta}\right)^{1/7} [1 - C_2\,(y/\delta)^2 + C_2^2\,(y/\delta)^4]\, d(y/\delta) = \frac{\sigma_c}{\sigma_{aw}} \cdot \frac{ms}{\delta}.$$

The left-hand side is a constant and assuming now that $\delta = B\,x\,Re^{-1/5}$ and $\mu_c = \mu_m$ the authors obtain

$$\eta = \frac{\sigma_{aw}}{\sigma_c} = K\left(\frac{x}{ms}\right)^{-0\cdot8} Re_c^{0\cdot2}. \quad (8)$$

In a graphical comparison of (8) with experiment the dependence of η on Re_c is omitted and a ± 40 per cent scatter noted. If $\mu_c \neq \mu_m$ and B is taken as $0{\cdot}37$ then (8) becomes

$$\eta = 3{\cdot}39 \left(\frac{x}{ms}\right)^{-0\cdot8} \left(Re_c \frac{\mu_c}{\mu_m}\right)^{0\cdot2}. \quad (9)$$

Rubesin [4] tackled the problem of the heat-transfer rate distribution along a flat plate with a step discontinuity in surface temperature. For a given unheated starting length L and wall temperature distribution $T_w(x)$ he found that the heat-transfer coefficient was

$$H(x, L) = \frac{q}{T_w - T_m} = 0{\cdot}0288 \frac{k}{x} Re_x^{4/5}\, Pr^{1/3} \left[1 - \left(\frac{L}{x}\right)^{39/40}\right]^{-7/39}. \quad (10)$$

His analysis used the integral form of the boundary-layer energy equation and among the assumptions were the following:

$$\frac{u}{u_m} = \left(\frac{y}{\delta}\right)^{1/7} \quad (11a)$$

$$\delta = 0{\cdot}37\, x\, Re_x^{-1/5} \quad (11b)$$

$$\theta = \frac{T - T_w}{T_m - T_w} = \left(\frac{y}{\delta_T}\right)^{1/7}. \quad (12)$$

Klein and Tribus [5] solved the inverse problem of finding $T_w(x)$ when the heat flux $q(x)$ is prescribed. They show that if H can be written as

$$H(x, L) = f(x)\,(x^\alpha - L^\alpha)^{-\beta}$$

then

$$T_w(x) - T_m = \int_{L=0}^{x} \frac{q(L)\,(x^\alpha - L^\alpha)^{\beta-1}\,\alpha\,.\,L^{\alpha-1}}{f(L)\,(-\beta)!\,(\beta-1)!}\, dL \quad (13)$$

Using the work of Rubesin we have

$\alpha = 39/40,\ \beta = 7/39,$

$$f(x) = 0{\cdot}0288 \frac{k}{x} Re_x^{4/5}\,.\,Pr^{1/3}\, x^{7/40}.$$

At Eckert's suggestion Klein and Tribus considered the particular example of a line heat source placed at the leading edge of a flat plate and calculated the variation of adiabatic wall temperature along the plate. This example is similar to the film cooling problem, the source simulating the heat released from the coolant slot. Putting

$$q(L)\, dL = \rho_c\, u_c\, s\, C_{p_c} (T_c - T_m)$$

at $x = 0$,

$$q = 0 \quad \text{for} \quad x > 0$$

and

$$L = 0$$

then equation (13) becomes

$$\eta = 5{\cdot}77 \left(\frac{C_{p_c}}{C_{p_m}}\right) Pr_m^{2/3} \left(\frac{x}{ms}\right)^{-0\cdot8} \left(Re_c \frac{c}{\mu m}\right)^{0\cdot2} \quad (14)$$

If $C_{p_c}/C_{p_m} = 1$ and $Pr_m = 0{\cdot}72$ then (14) reads

$$\eta = 4{\cdot}62 \left(\frac{x}{ms}\right)^{-0\cdot8} \left(Re_c \frac{\mu_c}{\mu_m}\right)^{0\cdot2} \quad (15)$$

In a more recent analysis Seban and Back [6] note the similarity between the approximate form of Wieghardt's temperature profile

$$\sigma/\sigma_{aw} = \exp[-C_2(y/\delta)^2] \quad \text{(see page 57)} \quad (7)$$

and a particular solution of the heat conduction equation,

$$\sigma/\sigma_w = \exp[-y^2/4\xi] \quad \text{where} \quad \xi = \int \alpha \, dt \quad (16)$$

The analogous equations are

$$\frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha(t)} \cdot \frac{\partial T}{\partial t} \quad \text{where} \quad \alpha = k/\rho C_p \quad (17)$$

and

$$\frac{\partial^2 T}{\partial y^2} = \frac{1}{\beta(x)} \cdot \frac{\partial T}{\partial t}, \quad \text{where} \quad \beta = k_e/\rho u C_p, \quad (18)$$

a rather gross contraction of the energy equation. Seban and Back, relying on the work of Hinze [6], take

$$\beta(x) = 0{\cdot}7\, \delta_2 \sqrt{(C_f/2)}$$
$$\delta_2 = 0{\cdot}036\, x\, Re_x^{-1/5}$$
$$C_f/2 = 0{\cdot}0296\, Re_x^{-1/5}$$

Thus $\beta \sim x^{0\cdot7}$ and from the analogy we have $\delta^2 \sim \int \beta \, dx$, i.e.

$$\delta \sim x^{0\cdot85}$$

Use of the conservation of energy, equation (3), then leads to the final result

$$\eta = 11{\cdot}2 \left(\frac{x}{ms}\right)^{-0\cdot85} \left(Re_c \frac{\mu_c}{\mu_m}\right)^{0\cdot15} \quad (19)$$

There is one other theoretical model [8] for the film cooling process which, in contrast to the previous papers, begins by assuming that the coolant film exists as a discreet layer (no mixing). Two empirical modifications are then made to take care of the mixing phenomena. As might be expected this model provides a good basis for correlating experimental data [8] taken reasonably close to the coolant slot, $(0 < x/s < 150)$. In view of the empirical nature of the modifications to the initial theory this model is not included here.

A MORE SIMPLE ANALYSIS

The assumptions here are:

(i) the flow is boundary-layer-like and $\delta = 0{\cdot}37\, x\, Re_x^{-1/5}$

(ii) the coolant and mainstream gases have the same composition

(iii) the temperature differences are small enough for C_p to be constant

(iv) the pressure is constant throughout the flow field

(v) the temperature and velocity boundary layers have the same thickness

(vi) the stagnation enthalpy is uniform across any section therefore $T_o = f(x)$ only

(vii) the mass-velocity power law holds with $n = 1/7$, i.e.

$$\frac{\rho u}{\rho_m u_m} = \left(\frac{y}{\delta}\right)^{1/7} \quad (20)$$

From an energy (total enthalpy) balance with an adiabatic wall and $C_{p_m} = C_{p_c} = C_p$ we have

$$\dot{m}_{bl} C_p T_{aw} = \dot{m}_c C_p T_{oc} + \dot{m}_e C_p T_{om} \quad (21)$$

The mass flow rate in the boundary layer is

$$\dot{m}_{bl} = \int_0^\delta \rho u \, dy$$

which from (20) gives

$$\dot{m}_{bl} = (7/8)\, \rho_m u_m \delta$$

The mass entrained in the boundary layer $\dot{m}_e$ is then

$$\dot{m}_e = \dot{m}_{bl} - \dot{m}_c$$

where $\dot{m}_c$ is the coolant mass flow rate

$$\dot{m}_c = \rho_c u_c s.$$

Substitution in (21) gives immediately

$$\eta' = \frac{h_{aw} - h_{om}}{h_{oc} - h_{om}} = \frac{8}{7} \cdot \frac{\rho_c u_c s}{\rho_m u_m \delta}.$$

Using assumption (i) and introducing a slot Reynolds number as $Re_c = (\rho_c u_c s/\mu_c)$ we obtain

$$\eta' = 3{\cdot}09 \left(\frac{x}{ms}\right)^{-0\cdot8} \left(Re_c \frac{\mu_c}{\mu_m}\right)^{0\cdot2} \quad (22)$$

Foreign-gas injection

This simple analysis may be extended to cover the case of foreign gas injection by relaxing assumption (iii) and substituting

$$C_p(x) = \frac{\dot{m}_e C_{p_m} + \dot{m}_c C_{p_c}}{\dot{m}_{bl}} \quad (23)$$

The result then is

$$\left.\begin{aligned} \eta' &= \frac{C_p T_{aw} - C_{p_m} T_{om}}{C_{p_c} T_{oc} - C_{p_m} T_{om}} = \\ & \frac{1}{1 + \dfrac{C_{p_c}}{C_{p_m}} \left\{ \dfrac{T_{oc} - T_{aw}}{T_{aw} - T_{om}} \right\}} = \\ & 3{\cdot}09 \left(\frac{x}{ms} \right)^{-0{\cdot}8} \times \left(Re_c \frac{\mu_c}{\mu_m} \right)^{0{\cdot}2} \end{aligned}\right\} \quad (24)$$

This may be rearranged to read

$$\eta'' = \frac{T_{aw} - T_{om}}{T_{oc} - T_{om}} = \frac{C_3 \, \eta'}{1 + (C_3 - 1)\, \eta'}$$

$$\text{where} \quad C_3 = C_{p_c}/C_{p_m} \qquad (25)$$

and η' is given by (22) or (24). For $C_3 = 1$, η' and η'' are identical. For $C_3 \neq 1$ equation (25) shows that, for large x/ms, $\eta'' \to C_3\eta'$. A result indicating the benefit of using a coolant with a large specific heat.

A comparison of the theoretical results

All but one [8] of the theories mentioned above are based on a boundary-layer model of the flow. They necessarily assume mixing of the coolant with the mainstream and are strictly asymptotic solutions valid only for large x/s. The precise definition of "large" can be found by correlating the experimental data. This is done later in the paper. It is interesting just to compare the theoretical results of Wieghardt, Hartnett *et al.* Tribus and Klein with the simple analysis presented here. The relevant expressions are

$$\eta = \eta' = 5{\cdot}44\,(\bar{x})^{-0{\cdot}8} \qquad \text{(reference 2)}$$

$$\eta = \eta' = 3{\cdot}39\,(\bar{x})^{-0{\cdot}8} \qquad \text{(reference 3)}$$

$$\eta = \eta' = 4{\cdot}62\,(\bar{x})^{-0{\cdot}8} \qquad \text{(reference 5)}$$

$$\eta' = 3{\cdot}09\,(\bar{x})^{-0{\cdot}8} \qquad \text{(this paper)}$$

where

$$\bar{x} = (x/ms)\,(Re_c\,\mu_c/\mu_m)^{-1/4}$$

For the case of foreign gas injection

$$\eta = 4{\cdot}62\,(C_{p_c}/C_{p_m})\,(\bar{x})^{-0{\cdot}8} \qquad \text{(reference 5)}$$

$$\eta' = 3{\cdot}09\,(\bar{x})^{-0{\cdot}8} \qquad \text{(this paper)}$$

where

$$\eta = \frac{C_{p_c}}{C_{p_m}}\eta' \bigg/ \left[1 + \left(\frac{C_{p_c} - C_{p_m}}{C_{p_m}}\right)\eta'\right]$$

provided

$$T_o \simeq T.$$

The striking similarity between the various analyses is not surprising since they all use the 1/7th power law similarity solution for the velocity profile together with an energy balance equation. The different constants result from differing approximations to the temperature profile. For example Wieghardt obtains an exponential form, Rubesin assumes a 1/7th power law temperature profile and in the simple analysis of this paper the total temperature T_o is assumed constant across the boundary layer.

COMPARISON WITH SOME EXPERIMENTAL DATA

A selection of the available experimental data has been made covering the widest ranges of slot Reynolds number, density ratio ρ_c/ρ_m and velocity ratio u_c/u_m, and correlated using equations (22) and (24), in an attempt to test the validity of these equations. In particular it is important to know over what range of

$$m(= \rho_c u_c/\rho_m u_m)$$

a boundary-layer model of the flow is likely to be valid.

Data [2, 3, 9] obtained at three widely different slot Reynolds numbers are presented in Fig. 2(a) as a plot of η' against x/ms. Hartnett *et al.* [3] produced a similar picture and noted a ± 40 per cent scatter about their own experimental data. Figure 2(b) demonstrates now the three sets of data chosen are brought closer together by plotting η' against $\bar{x}$ as suggested by most of the theoretical treatments. The final slope of the best curve through the data correlated in this way is close to $-0{\cdot}8$ but the data suggests a value of the constant in equation (22) rather larger than 3·09. A further comparison is made in Fig. 3 using the data of Wieghardt [2] and Seban [9]. For $0 \leqslant m \leqslant 1{\cdot}5$ their data are all expressible in the form $\eta' = A(\bar{x})^{-0{\cdot}8}$. Table 1 and Fig. 3 give the value of the constant A and show that for $m \leqslant 1$ the constant is nearer 3·7. Beyond $m = 1$

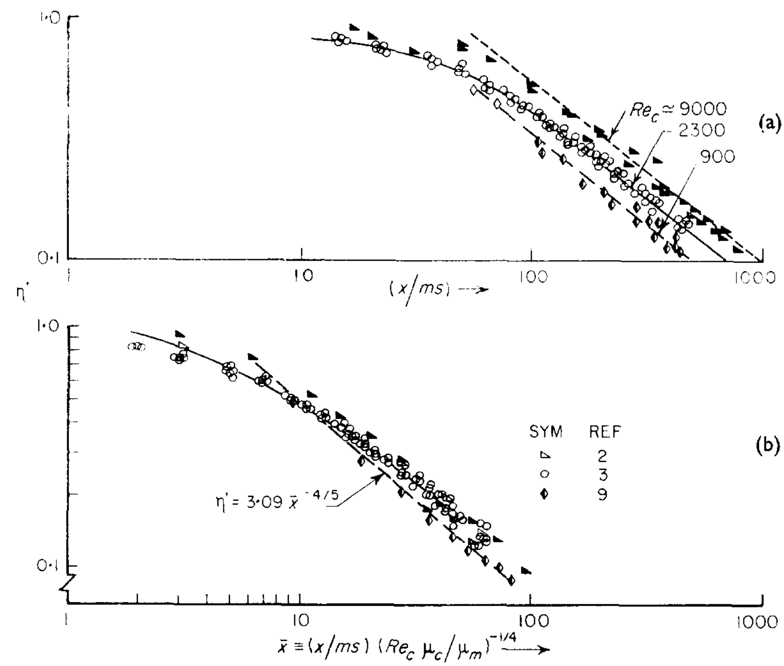

FIG. 2. The improvement of data correlation by inclusion of slot Reynolds number.

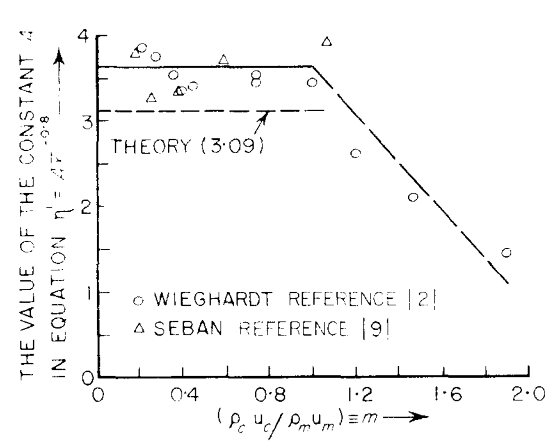

FIG. 3. The value of the constant A in the equation $\eta' = A(\bar{x})^{-4/5}$.

Table 1. The value of the constant A in the expression $\eta' = A\bar{x}^{-0{\cdot}8}$

Ref. and Symbol	m	$Re_1(\mu_c/\mu_m)$	A
Wieghardt [2]	0·22	5300	3·85
●	0·28	7400	3·72
	0·36	8900	3·92
	0·40	11500	3·33
	0·40	5000	3·33
	0·45	1200	3·40
	0·74	9100	3·52
	0·74	9900	3·48
	1·01	12500	3·48
	1·20	12200	2·62
Seban [9]	1·45	12800	2·10
▲	1·90	13100	1·48
	0·18	7100	3·78
	0·26	1500	3·28
	0·39	3100	3·37
	0·59	7250	3·71
	1·08	6100	3·93

the constant falls rapidly and for $m > 1{\cdot}5$ the slope of the experimental curve changes as shown in Fig. 4, indicating the inadequacy of the boundary-layer model for this range of m.

The data of Chin *et al.* [10] for single and multiple slots were correlated empirically in the original paper. They are replotted in the way suggested by equation (22) in Fig. 5. Although the data do not fit the equation both the single and the ten-slot data correlate very well using the ordinates appropriate to the turbulent

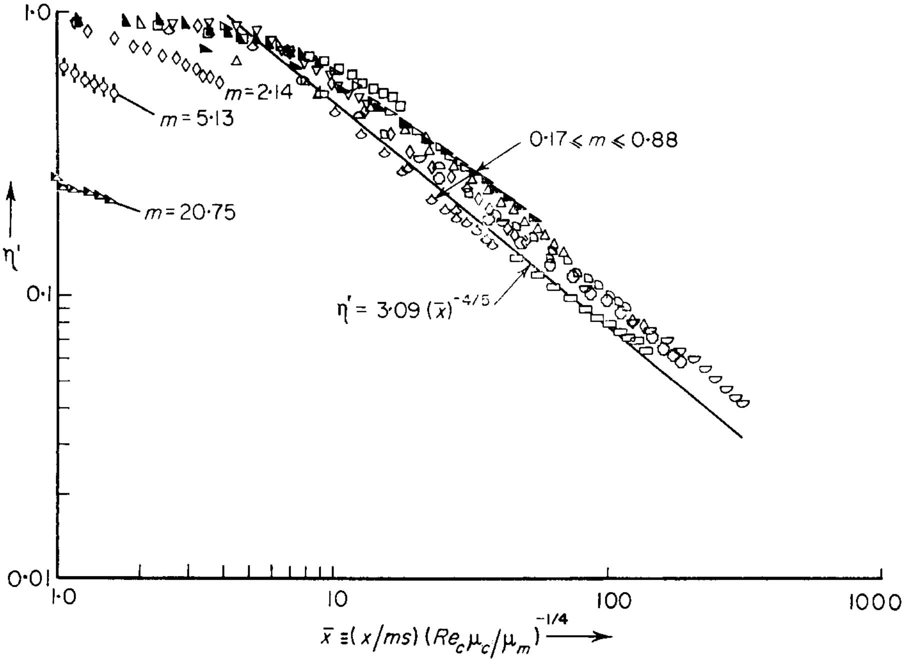

FIG. 4. Data taken from reference 9 to show the range of m for which the boundary-layer model is useful.

boundary-layer model. For the multi-slot case x is measured from the last slot and s is the height of an equivalent single slot through which all the cooling fluid passes. Finally the 20 row louvre data of Chin *et al.* is replotted, Fig. 5(b); again correlation is good. The negative slope of the correlated data tends towards 0·8, then decreases again for large (x/ms) because, according to reference 10, there was heat conduction through the plate.

In all the experimental data considered so far the density differences, temperature differences and flow velocities have been small so that ideas based on incompressible turbulent boundary layer theory might be expected to succeed. It is possible that the simple analysis given here may be useful under compressible flow conditions since experiments [12] have shown that the velocity profile on a flat adiabatic plate appears to be almost independent of Mach number and that the formula $\delta = 0{\cdot}37\, x\, Re_x^{-1/5}$ is usable up to $M = 2{\cdot}5$ at least. The highest Mach number data currently available is that of references 8 and 11. In reference 11 the values of η' are not unity at $x = 0$, the reason being [13] that the coolant temperature was measured well upstream of the slot and there was appreciable heat transfer between the measuring station and the slot exit. The results have therefore been corrected† and plotted in Fig. 6, they correlate very well on the basis of equation (22). The highest freestream Mach number used in the tests was 0·78. The coolant and freestream gas was air. In a later series of tests [8] helium was used as the coolant with injection velocities up to 3680 ft/s, ($M_c \simeq 1$). Again the highest freestream Mach number was approximately 0·8. The data selected from reference 8 covered the whole range of test variables including the highest value of m (i.e. 1·57). At this value of m the velocity ratio u_c/u_m was 3·83. The results are compared with equation (24) in Fig. 7. Considering the simplicity of the analysis for foreign gas injection the agreement is encouraging but not good enough for design purposes. A more realistic approach is obviously needed for the compressible turbulent boundary-layer flow of a gas mixture.

† $\eta'(x)$ corrected $= \eta'(x)/\eta'(x = 0)$.

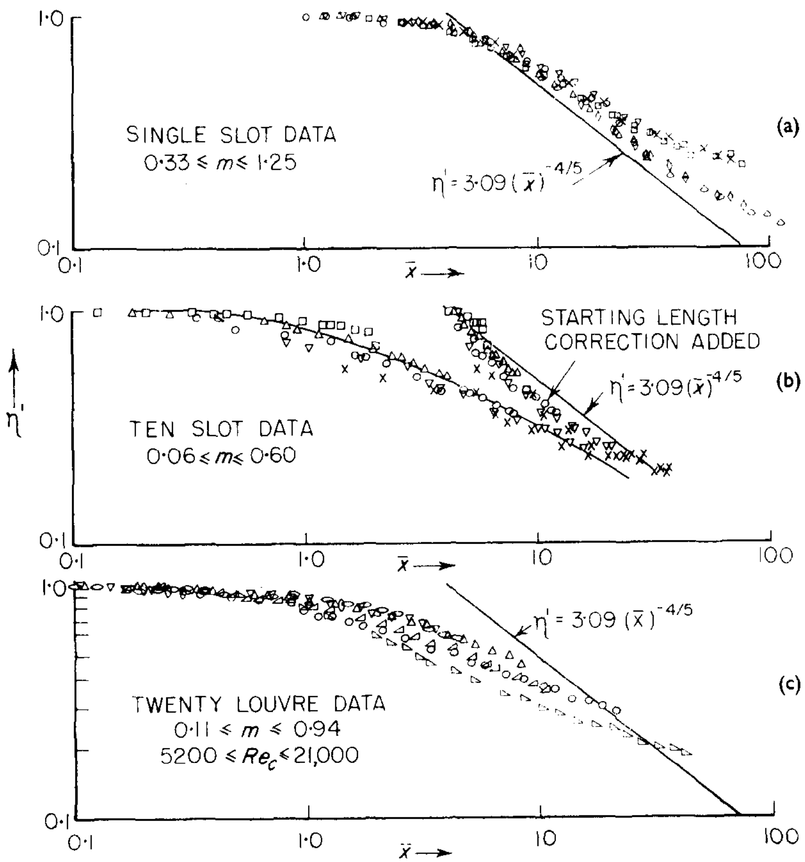

FIG. 5. The correlation of single, multi-slot and louvre data from reference 10.

POSSIBLE EXTENSIONS OF THE ANALYSIS

Corrections for finite δ at $x = 0$

Clearly the analyses presented here are only valid for large x/s. A "new" boundary layer is assumed to grow from the slot exit and to have zero thickness there. It should be possible to improve the analysis by taking account of the slot mass flow by equating one of its characteristics to that of a fictitious boundary layer growing from a point at a distance x_o ahead of the slot. For example, equating the mass flow in the fictitious boundary layer to that coming from the slot gives

$$\rho_c u_c s = \int_0^\delta \rho u \, dy = \rho_m u_m \int_0^1 (y/\delta)^{-1/7} \, d(y/\delta) = = \tfrac{7}{8} \rho_m u_m \delta$$

Putting $\delta = 0{\cdot}37\, x_o\, Re_{x_o}^{-1/5}$ gives on re-arrangement

$$\frac{x_o}{ms}\left(Re_c \frac{\mu_c}{\mu_m}\right)^{1/4} = 4{\cdot}1 \qquad (26)$$

Adopting this correction in the analysis changes the relation (22) to

$$\eta' = 3{\cdot}09\,[\bar{x} + 4{\cdot}1]^{-0{\cdot}8} \qquad (27)$$

This result has the additional advantage that $\eta' \to 1$ as $x \to 0$. The correction is easy to apply and undoubtedly improves the agreement with "theory" as demonstrated in Fig. 5(b). However much useful experimental data in the region $0 < \bar{x} < 10$ is effectively "lost" if (27) is used moreover the *correlation* is not improved. The aim here is to demonstrate that the parameter $\bar{x}$ has theoretical justification and practical utility

KEY

SYM	u_c/u_m	m	s
○	0·232	0·35	0·26 in.
△	0·328	0·53	↓
▽	0·450	0·76	
⊳	0·063	0·08	
⊲	0·120	0·16	
⬭	0·143	0·20	
0	0·200	0·28	
X	0·268	0·39	
+	0·372	0·56	
●	0·455	0·73	
⊖	0·535	0·85	
⦶	0·740	1·24	↓

KEY

SYM	u_c/u_m	m	s
D	0·332	0·56	0·135 in.
▽	0·543	0·77	↓
◖	0·715	1·05	
◠	0·815	1·25	↓
◊	0·220	0·29	0·5 in.
□	0·266	0·32	↓
◇	0·306	0·41	
△	0·410	0·58	
◊	0·55	0·78	
◊	0·33	0·46	↓
◇	0·46	0·64	0·135 in.
▷	0·55	0·79	↓
◁	0·75	1·12	↓

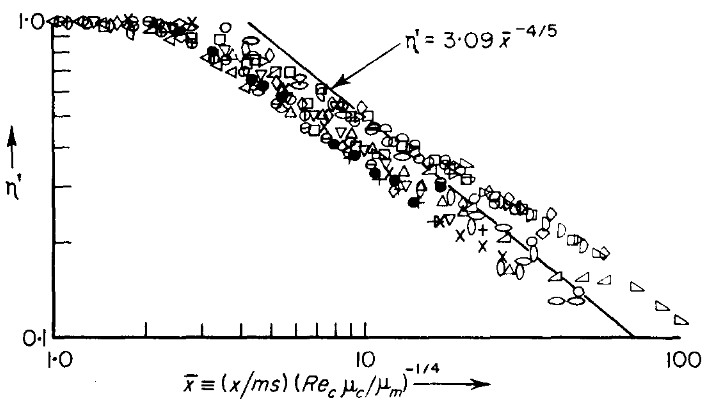

FIG. 6. The data of Papell and Trout [11] for air injection into a high temperature air mainstream.

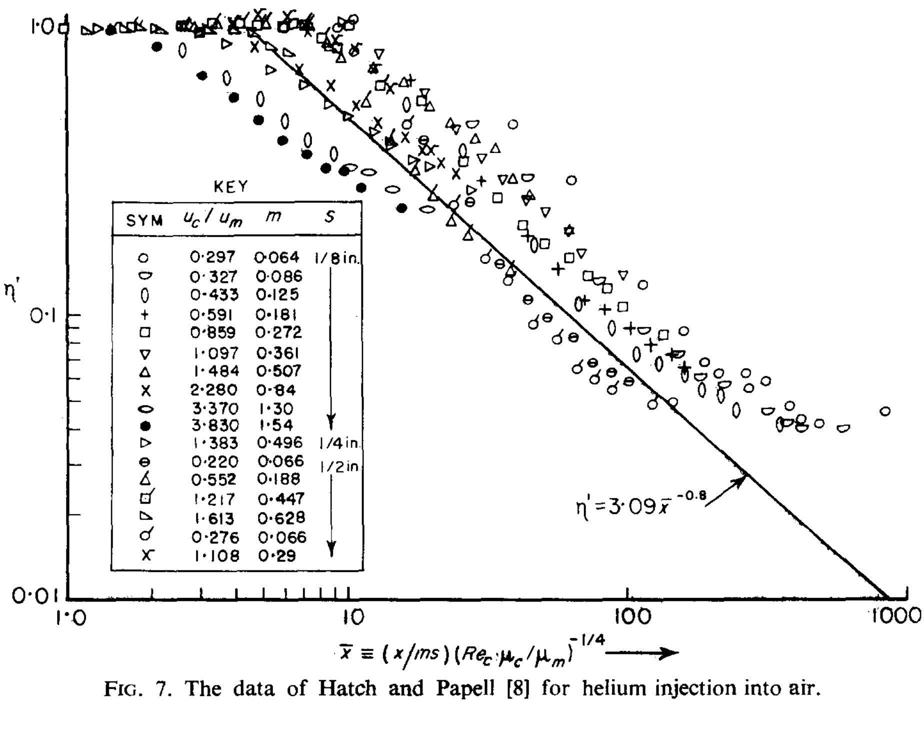

FIG. 7. The data of Hatch and Papell [8] for helium injection into air.

for correlating film cooling data, rather than to obtain the best possible fit between experiment and theory. The temptation to apply corrections has therefore been resisted.

The effect of pressure gradient

The simple analysis given here can obviously be extended to non-zero pressure gradient conditions provided $\delta(x)$ can be calculated. Numerous methods of calculation exist and in a correlation of some of these Stratford and Beavers [14] suggest

$$\left.\begin{aligned} &\delta = 0\cdot37\, X\, Re_X^{-1/5} \\ \text{where}\quad &X = P^{-1}\int_0^x P\,\mathrm{d}x \\ \text{and}\quad &P = \left[M\Big/\left(1+\frac{\gamma-1}{2}M^2\right)\right]^4 \end{aligned}\right\} \quad (28)$$

for free stream Reynolds numbers of the order of 10^6. The substitution of X for x in the parameter $\bar{x}$ should then correlate film cooling data obtained in the presence of a pressure gradient.

Heat transfer

Hartnett *et al.* [3] have shown that the standard solid-wall heat-transfer relations between Stanton number and Re_x may be used for predicting the heat transfer in the presence of film cooling provided that

(a) the coefficient of film cooling is defined as

$$H = \frac{q}{T_w - T_{aw}} \quad (29)$$

where T_{aw} is now the local value determined, for example, from equation (22),

(b) the properties used to evaluate the Reynolds number and Stanton number are evaluated at the reference temperature T^* where

$$T^* = 0\cdot5\,(T_w + T_m) + 0\cdot22\,(T_{aw} - T_m) \quad (30)$$

Temperatures T_w and T_{aw} are again local values.

Seban [9] defined H in the same way and provided m was less than one he obtained good agreement with the Colburn equation

$$\frac{H}{\rho_m\, u_m\, C_p} = 0\cdot37\, Re_x^{-1/5} \quad (31)$$

for $x/s \geqslant 50$. These two pieces of data suggest that a knowledge of the adiabatic wall temperature plus the use of Eckert's reference enthalpy method might lead to a solution of the film-cooled-wall with heat-transfer problem. More experimental data are urgently needed to confirm or destroy this suggestion.

CONCLUSIONS

The boundary-layer model of a film-cooled surface has been used by a number of authors to derive an analytical expression for effectiveness in the form

$$\eta' = \text{const.}\left\{\frac{x}{ms}\cdot\left(Re_c\frac{\mu_c}{\mu_m}\right)^{-1/4}\right\}^{-0\cdot8}$$

The present paper derives this result in a far simpler manner than has been used before. The paper emphasizes that the effectiveness should be based on enthalpy rather than temperature and that if this is done the same expression is theoretically valid for foreign gas injection.

The boundary-layer model correlates a wide range of data provided $0 \leqslant m \leqslant 1\cdot5$. It can be used for multi-slot or louvre cooling by postulating a single equivalent slot.

The analysis can be extended very simply to cover the effects of pressure gradient and heat transfer. More experimental data are needed to test the validity of such extensions.

It must be emphasized that the boundary-layer model is strictly an asymptotically correct solution of the film cooling problem. As such its accuracy improves as x/s increases and at infinity it must hold no matter what the initial conditions. However, the model is clearly inadequate for $m > 1\cdot5$ over the range of x/s currently of interest, and inaccurate close to the coolant slot when there are large density differences between the coolant and mainstream gases.

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Résumé—Le modèle de la couche limite pour corréler les données expérimentales sur les parois adiabatiques refroidies par film est réexaminé. Les analyses existantes sont passées en revue. Une approche beaucoup plus simple est donnée qui peut aussi couvrir le cas de l'injection d'un gaz étranger. L'utilité et les limitations du modèle de la couche limite sont mises en relief par comparaison avec les données expérimentales.

Zusammenfassung—Das Grenzschichtmodell, das experimentell ermittelte Werte mit Film-gekühlten adiabaten Wänden in Beziehung setzt, wird überprüft. Bestehende Analysen dazu werden durchgeseshen. Es wird eine viel einfachere Näherung gegeben, die auch den Fall der Fremdgaseinspritzung erfassen kann. Die Grenzen und die Verwendbarkeit des Grenzschichtmodells wird durch einen Vergleich mit experimentellen Ergebnissen gezeigt.

Аннотация—Вновь анализируется модель пограничного слоя для корреляции экспериментальных данных для стенок при пленочном охлаждении в адиабатических условиях. Снова рассматриваются существующие анализы. Дается более простой подход, который также включает случай вдува инородного газа. Применение и ограничения модели пограничного слоя показаны в сравнении с экспериментальными данными.